Chapter 22 – Comparing Two Proportions

1. **Online social networking.**

   It is very unlikely that samples would show an observed difference this large if, in fact, there was no real difference in the proportions of boys and girls who have used online social networks.

2. **Science news.**

   If, in fact, there is no difference in the proportion of the population who get science news from the Internet and from TV, then it's not unusual to observe a difference of 1% by sampling.

3. **Name recognition.**

   The ads may be effective. If there had been no real change in the name recognition, there would only be about a 3% chance the percentage of voters who had heard of this candidate would be at least this much higher in a different sample.

4. **Origins.**

   There is no evidence of a change in opinion. Even if there's been no change in public opinion, there is a 37% chance we would see this much difference, or more, from one sample to another.

5. **Revealing information.**

   This test is not appropriate for these data, since the responses are not from independent groups, but are from the same individuals. The independent samples condition has been violated.

6. **Regulating access.**

   The 790 parents are a subset of the 935 parents, so the two groups are not independent. This violates the independent samples condition.

7. **Gender gap.**

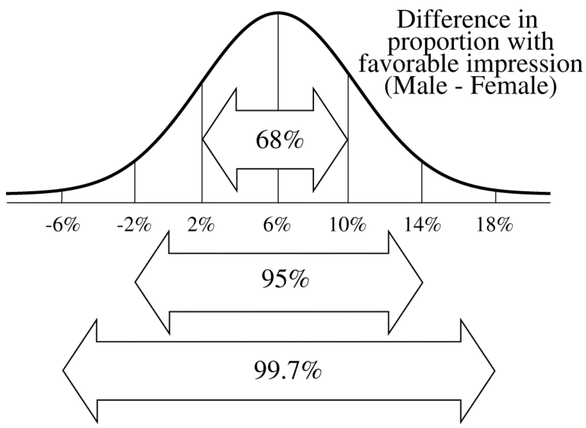
   a) This is a stratified random sample, stratified by gender.

   b) We would expect the difference in proportions in the sample to be the same as the difference in proportions in the population, with the percentage of respondents with a favorable impression of the candidate 6% higher among males.

   c) The standard deviation of the difference in proportions is:

$$\sigma(\hat{p}_M - \hat{p}_F) = \sqrt{\frac{\hat{p}_M \hat{q}_M}{n_M} + \frac{\hat{p}_F \hat{q}_F}{n_F}} = \sqrt{\frac{(0.59)(0.41)}{300} + \frac{(0.53)(0.47)}{300}} \approx 4\%$$

d)



e) The campaign could certainly be misled by the poll. According to the model, a poll showing little difference could occur relatively frequently. That result is only 1.5 standard deviations below the expected difference in proportions.

8. Buy it again?

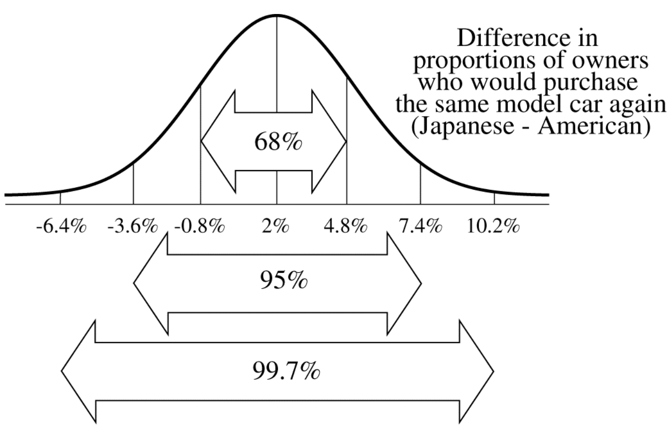
a) This is a stratified random sample, stratified by country of origin of the car.

b) We would expect the difference in proportions in the sample to be the same as the difference in proportions in the population, with the percentage of respondents who would purchase the same model again 2% higher among owners of Japanese cars than among owners of American cars.

c) The standard deviation of the difference in proportions is:

$$\sigma(\hat{p}_J - \hat{p}_A) = \sqrt{\frac{\hat{p}_J\hat{q}_J}{n_J} + \frac{\hat{p}_A\hat{q}_A}{n_A}} = \sqrt{\frac{(0.78)(0.22)}{450} + \frac{(0.76)(0.24)}{450}} \approx 2.8\%$$

d)



e) The magazine could certainly be misled by the poll. According to the model, a poll showing greater satisfaction among owners of American cars could occur relatively frequently. That result is less than one standard deviation below the expected difference in proportions.

9. Arthritis.

a) **Randomization condition:** Americans age 65 and older were selected randomly.
10% condition: 1012 men and 1062 women are less than 10% of all men and women.
Independent samples condition: The sample of men and the sample of women were drawn independently of each other.
Success/Failure condition: $n\hat{p}$(men) = 411, $n\hat{q}$(men) = 601, $n\hat{p}$(women) = 535, and $n\hat{q}$(women) = 527 are all greater than 10, so the samples are both large enough.

Since the conditions have been satisfied, we will find a two-proportion z-interval.

b) $\left(\hat{p}_F - \hat{p}_M\right) \pm z^* \sqrt{\dfrac{\hat{p}_F\hat{q}_F}{n_F} + \dfrac{\hat{p}_M\hat{q}_M}{n_M}} = \left(\frac{535}{1062} - \frac{411}{1012}\right) \pm 1.960\sqrt{\dfrac{\left(\frac{535}{1062}\right)\left(\frac{527}{1062}\right)}{1062} + \dfrac{\left(\frac{411}{1012}\right)\left(\frac{601}{1012}\right)}{1012}} = (0.055, 0.140)$

c) We are 95% confident that the proportion of American women age 65 and older who suffer from arthritis is between 5.5% and 14.0% higher than the proportion of American men the same age who suffer from arthritis.

d) Since the interval for the difference in proportions of arthritis sufferers does not contain 0, there is strong evidence that arthritis is more likely to afflict women than men.

10. Graduation.

a) **Randomization condition:** Assume that the samples are representative of all recent graduates.
10% condition: Although large, the samples are less than 10% of all graduates.
Independent samples condition: The sample of men and the sample of women were drawn independently of each other.
Success/Failure condition: The samples are very large, certainly large enough for the methods of inference to be used.

Since the conditions have been satisfied, we will find a two-proportion *z*-interval.

b) $\left(\hat{p}_F - \hat{p}_M\right) \pm z^* \sqrt{\dfrac{\hat{p}_F\hat{q}_F}{n_F} + \dfrac{\hat{p}_M\hat{q}_M}{n_M}}$

$= (0.881 - 0.849) \pm 1.960\sqrt{\dfrac{(0.881)(0.119)}{12{,}678} + \dfrac{(0.849)(0.151)}{12{,}460}} = (0.024, 0.040)$

c) We are 95% confident that the proportion of 24-year-old American women who have graduated from high school is between 2.4% and 4.0% higher than the proportion of American men the same age who have graduated from high school.

d) Since the interval for the difference in proportions of high school graduates does not contain 0, there is strong evidence that women are more likely than men to complete high school.

11. Pets.

a) $SE\left(\hat{p}_{Herb} - \hat{p}_{None}\right) = \sqrt{\dfrac{\hat{p}_{Herb}\hat{q}_{Herb}}{n_{Herb}} + \dfrac{\hat{p}_{None}\hat{q}_{None}}{n_{None}}} = \sqrt{\dfrac{\left(\frac{473}{827}\right)\left(\frac{354}{827}\right)}{827} + \dfrac{\left(\frac{19}{130}\right)\left(\frac{111}{130}\right)}{130}} = 0.035$

b) **Randomization condition:** Assume that the dogs studied were representative of all dogs.
10% condition: 827 dogs from homes with herbicide used regularly and 130 dogs from homes with no herbicide used are less than 10% of all dogs.
Independent samples condition: The samples were drawn independently of each other.
Success/Failure condition: $n\hat{p}$(herb) = 473, $n\hat{q}$(herb) = 354, $n\hat{p}$(none) = 19, and $n\hat{q}$(none) = 111 are all greater than 10, so the samples are both large enough.

Since the conditions have been satisfied, we will find a two-proportion z-interval.

$$\left(\hat{p}_{Herb} - \hat{p}_{None}\right) \pm z^* \sqrt{\frac{\hat{p}_{Herb}\hat{q}_{Herb}}{n_{Herb}} + \frac{\hat{p}_{None}\hat{q}_{None}}{n_{None}}}$$

$$= \left(\tfrac{473}{827} - \tfrac{19}{130}\right) \pm 1.960 \sqrt{\frac{\left(\frac{473}{827}\right)\left(\frac{354}{827}\right)}{827} + \frac{\left(\frac{19}{130}\right)\left(\frac{111}{130}\right)}{130}} = (0.356, 0.495)$$

c) We are 95% confident that the proportion of pets with a malignant lymphoma in homes where herbicides are used is between 35.6% and 49.5% higher than the proportion of pets with lymphoma in homes where no pesticides are used.

12. Carpal Tunnel.

a) $SE\left(\hat{p}_{Surg} - \hat{p}_{Splint}\right) = \sqrt{\frac{\hat{p}_{Surg}\hat{q}_{Surg}}{n_{Surg}} + \frac{\hat{p}_{Splint}\hat{q}_{Splint}}{n_{Splint}}} = \sqrt{\frac{(0.80)(0.20)}{88} + \frac{(0.54)(0.46)}{88}} = 0.068$

b) **Randomization condition:** It's not clear whether or not this study was an experiment. If so, assume that the subjects were randomly allocated to treatment groups. If not, assume that the subjects are representative of all carpal tunnel sufferers.
10% condition: 88 subjects in each group are less than 10% of all carpal tunnel sufferers.
Independent samples condition: The improvement rates of the two groups are not related.
Success/Failure condition: $n\hat{p}$ (surg) = (88)(0.80) = 70, $n\hat{q}$ (surg) = (88)(0.20) = 18, $n\hat{p}$ (splint) = (88)(0.54) = 48, and $n\hat{q}$ (splint) = (88)(0.46) = 40 are all greater than 10, so the samples are both large enough.

Since the conditions have been satisfied, we will find a two-proportion z-interval.

$$\left(\hat{p}_{Surg} - \hat{p}_{Splint}\right) \pm z^* \sqrt{\frac{\hat{p}_{Surg}\hat{q}_{Surg}}{n_{Surg}} + \frac{\hat{p}_{Splint}\hat{q}_{Splint}}{n_{Splint}}}$$

$$= (0.80 - 0.54) \pm 1.960 \sqrt{\frac{(0.80)(0.20)}{88} + \frac{(0.54)(0.46)}{88}} = (0.126, 0.394)$$

c) We are 95% confident that the proportion of patients who show improvement in carpal tunnel syndrome with surgery is between 12.6% and 39.4% higher than the proportion who show improvement with wrist splints.

13. Ear infections.

a) **Randomization condition:** The babies were randomly assigned to the two treatment groups.
Independent samples condition: The groups were assigned randomly, so the groups are not related.
Success/Failure condition: $n\hat{p}$ (vaccine) = 333, $n\hat{q}$ (vaccine) = 2122, $n\hat{p}$ (none) = 499, and $n\hat{q}$ (none) = 1953 are all greater than 10, so the samples are both large enough.

Since the conditions have been satisfied, we will find a two-proportion z-interval.

b) $\left(\hat{p}_{None} - \hat{p}_{Vacc}\right) \pm z^* \sqrt{\dfrac{\hat{p}_{None}\hat{q}_{None}}{n_{None}} + \dfrac{\hat{p}_{Vacc}\hat{q}_{Vacc}}{n_{Vacc}}}$

$$= \left(\tfrac{499}{2452} - \tfrac{333}{2455}\right) \pm 1.960 \sqrt{\frac{\left(\frac{499}{2452}\right)\left(\frac{1953}{2452}\right)}{2452} + \frac{\left(\frac{333}{2455}\right)\left(\frac{2122}{2455}\right)}{2455}} = \left(0.047, 0.089\right)$$

c) We are 95% confident that the proportion of unvaccinated babies who develop ear infections is between 4.7% and 8.9% higher than the proportion of vaccinated babies who develop ear infections. The vaccinations appear to be effective, especially considering the 20% infection rate among the unvaccinated. A reduction of 5% to 9% is meaningful.

14. Anorexia.

a) **Randomization condition:** The women were randomly assigned to the treatment groups.
Independent samples condition: The groups were assigned randomly, so the groups are not related.
Success/Failure condition: $n\hat{p}$(Prozac) = 35, $n\hat{q}$(Prozac) = 14, $n\hat{p}$(placebo) = 32, and $n\hat{q}$(placebo) = 12 are all greater than 10, so the samples are both large enough.

Since the conditions have been satisfied, we will find a two-proportion z-interval.

b) $\left(\hat{p}_{Proz} - \hat{p}_{Plac}\right) \pm z^* \sqrt{\dfrac{\hat{p}_{Proz}\hat{q}_{Proz}}{n_{Proz}} + \dfrac{\hat{p}_{Plac}\hat{q}_{Plac}}{n_{Plac}}}$

$$= \left(\tfrac{35}{49} - \tfrac{32}{44}\right) \pm 1.960 \sqrt{\frac{\left(\frac{35}{49}\right)\left(\frac{14}{49}\right)}{49} + \frac{\left(\frac{32}{44}\right)\left(\frac{12}{44}\right)}{44}} = \left(-0.20, 0.17\right)$$

c) We are 95% confident that the proportion of women taking Prozac deemed healthy is between 20% lower and 17% higher than the proportion women taking a placebo. Prozac does not appear to be effective, since 0 is in the confidence interval. There is no evidence of a difference in the effectiveness of Prozac and a placebo.

15. Another ear infection.

a) H_0: The proportion of vaccinated babies who get ear infections is the same as the proportion of unvaccinated babies who get ear infections.
$\left(p_{Vacc} = p_{None} \text{ or } p_{Vacc} - p_{None} = 0\right)$

H_A: The proportion of vaccinated babies who get ear infections is the lower than the proportion of unvaccinated babies who get ear infections.
$\left(p_{Vacc} < p_{None} \text{ or } p_{Vacc} - p_{None} < 0\right)$

b) Since 0 is not in the confidence interval, reject the null hypothesis. There is evidence that the vaccine reduces the rate of ear infections.

c) Since a 95% confidence interval was used originally, the alpha level is half of 5%, or 2.5%

d) If we think that the vaccine really reduces the rate of ear infections and it really does not reduce the rate of ear infections, then we have committed a Type I error.

e) Babies would be given ineffective vaccines.

16. Anorexia again.

a) H_0: The proportion of women taking Prozac who are deemed healthy is the same as the proportion of women taking the placebo who are deemed healthy.
$\left(p_{Prozac} = p_{Placebo} \text{ or } p_{Prozac} - p_{Placebo} = 0\right)$

H_A: The proportion of women taking Prozac who are deemed healthy is greater than the proportion of women taking the placebo who are deemed healthy.
$\left(p_{Prozac} > p_{Placebo} \text{ or } p_{Prozac} - p_{Placebo} > 0\right)$

b) Since 0 is in the confidence interval, fail to reject the null hypothesis. There is no evidence that Prozac is an effective treatment for anorexia.

c) Since a 95% confidence interval was used originally, the alpha level is half of 5%, or 2.5%

d) If we think that Prozac is not effective and it is, we have committed a Type II error.

e) We might overlook a potentially helpful treatment.

17. Teen smoking, part I.

a) This is a prospective observational study.

b) H_0: The proportion of teen smokers among the group whose parents disapprove of smoking is the same as the proportion of teen smokers among the group whose parents are lenient about smoking. $\left(p_{Dis} = p_{Len} \text{ or } p_{Dis} - p_{Len} = 0\right)$

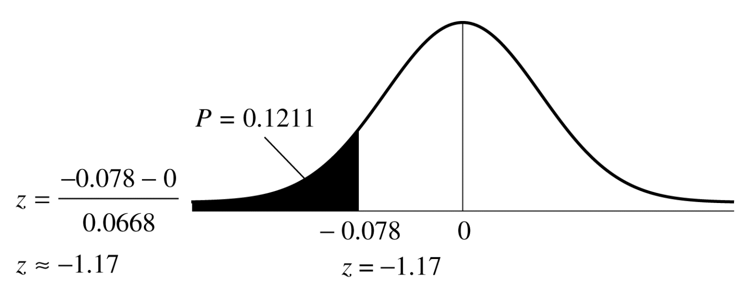
H_A: The proportion of teen smokers among the group whose parents disapprove of smoking is lower than the proportion of teen smokers among the group whose parents are lenient about smoking. $\left(p_{Dis} < p_{Len} \text{ or } p_{Dis} - p_{Len} < 0\right)$

c) **Randomization condition:** Assume that the teens surveyed are representative of all teens.
10% condition: 284 and 41 are both less than 10% of all teens.
Independent samples condition: The groups were surveyed independently.
Success/Failure condition: $n\hat{p}$(disapprove) = 54, $n\hat{q}$(disapprove) = 230, $n\hat{p}$(lenient) = 11, and $n\hat{q}$(lenient) = 30 are all greater than 10, so the samples are both large enough.

Since the conditions have been satisfied, we will model the sampling distribution of the difference in proportion with a Normal model with mean 0 and standard deviation estimated by $SE_{\text{pooled}}\left(\hat{p}_{Dis} - \hat{p}_{Len}\right) = \sqrt{\dfrac{\hat{p}_{\text{pooled}}\hat{q}_{\text{pooled}}}{n_{Dis}} + \dfrac{\hat{p}_{\text{pooled}}\hat{q}_{\text{pooled}}}{n_{Len}}} = \sqrt{\dfrac{\left(\frac{65}{325}\right)\left(\frac{260}{325}\right)}{284} + \dfrac{\left(\frac{65}{325}\right)\left(\frac{260}{325}\right)}{41}} = 0.0668$.

d) The observed difference between the proportions is 0.190 – 0.268 = – 0.078.

Since the *P*-value = 0.1211 is high, we fail to reject the null hypothesis. There is little evidence to suggest that parental attitudes influence teens' decisions to smoke.



e) If there is no difference in the proportions, there is about a 12% chance of seeing the observed difference or larger by natural sampling variation.

f) If teens' decisions about smoking *are* influenced, we have committed a Type II error.

18. Depression.

a) This is a prospective observational study.

b) H_0: The proportion of cardiac patients without depression who died within the 4 years is the same as the proportion of cardiac patients with depression who died during the same time period. $(p_{None} = p_{Dep} \text{ or } p_{None} - p_{Dep} = 0)$

H_A : The proportion of cardiac patients without depression who died within the 4 years is the less than the proportion of cardiac patients with depression who died during the same time period. $(p_{None} < p_{Dep} \text{ or } p_{None} - p_{Dep} < 0)$

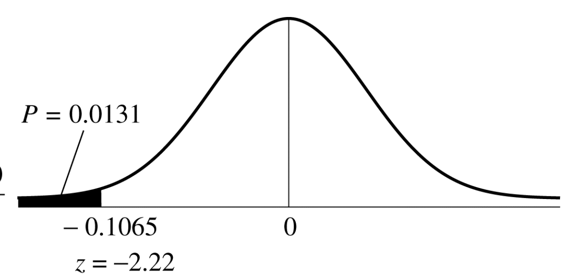
c) **Randomization condition:** Assume that the cardiac patients followed by the study are representative of all cardiac patients.
10% condition: 361 and 89 are both less than 10% of all teens.
Independent samples condition: The groups are not associated.
Success/Failure condition: $n\hat{p}$(no depression) = 67, $n\hat{q}$(no depression) = 294, $n\hat{p}$(depression) = 26, and $n\hat{q}$(depression) = 63 are all greater than 10, so the samples are both large enough.

Since the conditions have been satisfied, we will model the sampling distribution of the difference in proportion with a Normal model with mean 0 and standard deviation estimated by $SE_{\text{pooled}}(\hat{p}_{None} - \hat{p}_{Dep}) = \sqrt{\dfrac{\hat{p}_{\text{pooled}}\hat{q}_{\text{pooled}}}{n_{None}} + \dfrac{\hat{p}_{\text{pooled}}\hat{q}_{\text{pooled}}}{n_{Dep}}} = \sqrt{\dfrac{\left(\frac{93}{450}\right)\left(\frac{357}{450}\right)}{361} + \dfrac{\left(\frac{93}{450}\right)\left(\frac{357}{450}\right)}{89}} \approx 0.0479$.

d) The observed difference between the proportions is: 0.1856 – 0.2921 = – 0.1065.

Since the *P*-value = 0.0131 is low, we reject the null hypothesis. There is strong evidence to suggest that the proportion of non-depressed cardiac patients who die within 4 years is less than the proportion of depressed cardiac patients who die within 4 years.

$$z = \frac{-0.1065 - 0}{0.0479}$$
$$z \approx -2.22$$



e) If there is no difference in the proportions, we will see an observed difference this large or larger only about 1.3% of the time by natural sampling variation.

f) If cardiac patients without depression don't actually have a lower proportion of deaths in 4 years than cardiac patients with depression, then we have committed a Type I error.

19. Teen smoking, part II.

a) Since the conditions have already been satisfied in a previous exercise, we will find a two-proportion z-interval.

$$\left(\hat{p}_{Dis} - \hat{p}_{Len}\right) \pm z^* \sqrt{\frac{\hat{p}_{Dis}\hat{q}_{Dis}}{n_{Dis}} + \frac{\hat{p}_{Len}\hat{q}_{Len}}{n_{Len}}}$$

$$= \left(\tfrac{54}{284} - \tfrac{11}{41}\right) \pm 1.960\sqrt{\frac{\left(\frac{54}{284}\right)\left(\frac{230}{284}\right)}{284} + \frac{\left(\frac{11}{41}\right)\left(\frac{30}{41}\right)}{41}} = (-0.065, 0.221)$$

b) We are 95% confident that the proportion of teens whose parents disapprove of smoking who will eventually smoke is between 6.5% less and 22.1% more than for teens with parents who are lenient about smoking.

c) We expect 95% of random samples of this size to produce intervals that contain the true difference between the proportions.

20. Depression revisited.

a) Since the conditions have already been satisfied in a previous exercise, we will find a two-proportion z-interval.

$$\left(\hat{p}_{None} - \hat{p}_{Dep}\right) \pm z^* \sqrt{\frac{\hat{p}_{None}\hat{q}_{None}}{n_{None}} + \frac{\hat{p}_{Dep}\hat{q}_{Dep}}{n_{Dep}}}$$

$$= \left(\tfrac{67}{361} - \tfrac{26}{89}\right) \pm 1.960\sqrt{\frac{\left(\frac{67}{361}\right)\left(\frac{294}{361}\right)}{361} + \frac{\left(\frac{26}{89}\right)\left(\frac{63}{89}\right)}{89}} = (0.004, 0.209)$$

b) We are 95% confident that the proportion of cardiac patients who die within 4 years is between 0.4% and 20.9% higher for depressed patients than for non-depressed patients.

c) We expect 95% of random samples of this size to produce intervals that contain the true difference between the proportions.

21. Pregnancy.

a) This is not an experiment, since subjects were not assigned to treatments. This is an observational study.

b) H_0: The proportion of live births is the same for women under the age of 38 as it is for women 38 or older. $\left(p_{<38} = p_{\geq 38} \text{ or } p_{<38} - p_{\geq 38} = 0\right)$

H_A: The proportion of live births is different for women under the age of 38 than for women 38 or older. $\left(p_{<38} \neq p_{\geq 38} \text{ or } p_{<38} - p_{\geq 38} \neq 0\right)$

Randomization condition: Assume that the women studied are representative of all women.
10% condition: 157 and 89 are both less than 10% of all women.
Independent samples condition: The groups are not associated.
Success/Failure condition: $n\hat{p}$(under 38) = 42, $n\hat{q}$(under 38) = 115, $n\hat{p}$(38 and over) = 7, and $n\hat{q}$(38 and over) = 82 are not all greater than 10, since the observed number of live births is only 7. However, if we check the pooled value, $n\hat{p}_{\text{pooled}}$(38 and over) = (89)(0.1992) = 18. All of the samples are large enough.

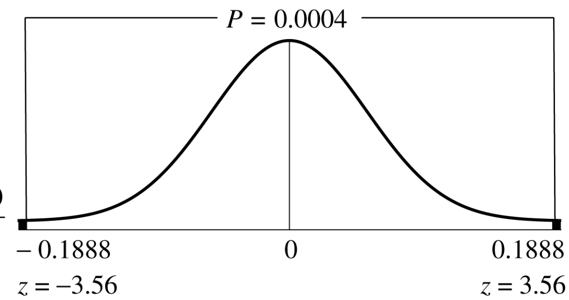
Since the conditions have been satisfied, we will model the sampling distribution of the difference in proportion with a Normal model with mean 0 and standard deviation estimated by $SE_{\text{pooled}}\left(\hat{p}_{<38} - \hat{p}_{\geq 38}\right) = \sqrt{\dfrac{\hat{p}_{\text{pooled}}\hat{q}_{\text{pooled}}}{n_{<38}} + \dfrac{\hat{p}_{\text{pooled}}\hat{q}_{\text{pooled}}}{n_{\geq 38}}} = \sqrt{\dfrac{\left(\frac{49}{246}\right)\left(\frac{197}{246}\right)}{157} + \dfrac{\left(\frac{49}{246}\right)\left(\frac{197}{246}\right)}{89}} \approx 0.0530$.

The observed difference between the proportions is: 0.2675 – 0.0787 = 0.1888.

Since the P-value = 0.0004 is low, we reject the null hypothesis. There is strong evidence to suggest a difference in the proportion of live births for women under 38 and women 38 and over at this clinic. In fact, the evidence suggests that women under 38 have a higher proportion of live births.

$$z = \frac{0.1888 - 0}{0.0530}$$

$$z \approx 3.56$$



c) $\left(\hat{p}_{<38} - \hat{p}_{\geq 38}\right) \pm z^* \sqrt{\dfrac{\hat{p}_{<38}\hat{q}_{<38}}{n_{<38}} + \dfrac{\hat{p}_{\geq 38}\hat{q}_{\geq 38}}{n_{\geq 38}}}$

$$= \left(\tfrac{42}{157} - \tfrac{7}{89}\right) \pm 1.960\sqrt{\frac{\left(\frac{42}{157}\right)\left(\frac{115}{157}\right)}{157} + \frac{\left(\frac{7}{89}\right)\left(\frac{82}{89}\right)}{89}} = (0.100, 0.278)$$

We are 95% confident that the proportion of live births for patients at this clinic is between 10.0% and 27.8% higher for women under 38 than for women 38 and over. However, the Success/Failure condition is not met for the older women, so we should be cautious when using this interval. (The expected number of successes from the pooled proportion cannot be used for a condition for a confidence interval. It's based upon an assumption that the proportions are the same. We don't make that assumption in a confidence interval. In fact, we are implicitly assuming a *difference*, by finding an interval for the difference in proportion.)

22. Birthweight.

a) This is not an experiment, since subjects were not assigned to treatments. This is an observational study.

b) H_0: The proportion of low birthweight is the same. $\left(p_{Exp} = p_{Not} \text{ or } p_{Exp} - p_{Not} = 0\right)$

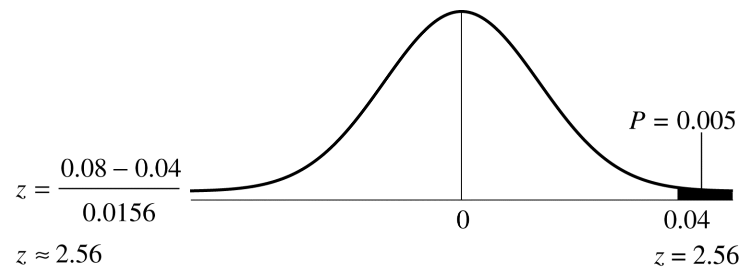
H_A: The proportion of low birthweight is higher for women exposed to soot and ash. $\left(p_{Exp} > p_{Not} \text{ or } p_{Exp} - p_{Not} > 0\right)$

Randomization condition: Assume that the women are representative of all women.
10% condition: 182 and 2300 are both less than 10% of all women.
Independent samples condition: The groups don't appear to be associated, with respect to soot and ash exposure, but all of the women were in New York. There may be confounding variable explaining any relationship between exposure and birthweight.
Success/Failure condition: $n\hat{p}$(Exposed) = 15, $n\hat{q}$(Exposed) = 167, $n\hat{p}$(Not) = 92, and $n\hat{q}$(Not) = 2208 are all greater than 10. All of the samples are large enough.

Since the conditions have been satisfied, we will model the sampling distribution of the difference in proportion with a Normal model with mean 0 and standard deviation estimated by $SE_{\text{pooled}}\left(\hat{p}_{Exp}-\hat{p}_{Not}\right)=\sqrt{\frac{\hat{p}_{\text{p}}\hat{q}_{\text{p}}}{n_{Exp}}+\frac{\hat{p}_{\text{p}}\hat{q}_{\text{p}}}{n_{Not}}}=\sqrt{\frac{\left(\frac{107}{2482}\right)\left(\frac{2375}{2482}\right)}{182}+\frac{\left(\frac{107}{2482}\right)\left(\frac{2375}{2482}\right)}{2300}}\approx 0.0156$.

The observed difference between the proportions is: 0.08 – 0.04 = 0.04.

Since the P-value = 0.005 is low, we reject the null hypothesis. There is strong evidence that the proportion of low birthweight babies is higher in the women exposed to soot and ash after the World Trade Center attacks.



c) $\left(\hat{p}_{Exp}-\hat{p}_{Not}\right)\pm z^{*}\sqrt{\frac{\hat{p}_{Exp}\hat{q}_{Exp}}{n_{Exp}}+\frac{\hat{p}_{Not}\hat{q}_{Not}}{n_{Not}}}$

$$=\left(\frac{15}{182}-\frac{92}{2300}\right)\pm 1.960\sqrt{\frac{\left(\frac{15}{182}\right)\left(\frac{167}{182}\right)}{182}+\frac{\left(\frac{92}{2300}\right)\left(\frac{2208}{2300}\right)}{2300}}=\left(0.002,\ 0.083\right)$$

We are 95% confident that the proportion of low birthweight babies is between 0.2% and 8.3 % higher for mothers exposed to soot and ash after the World Trade Center attacks, than the proportion of low birthweight babies for mothers not exposed.

23. Politics and sex.

a) H_0: The proportion of voters in support of the candidate is the same before and after news of his extramarital affair got out. $\left(p_B = p_A \text{ or } p_B - p_A = 0\right)$

H_A: The proportion of voters in support of the candidate has decreased after news of his extramarital affair got out. $\left(p_B > p_A \text{ or } p_B - p_A > 0\right)$

Randomization condition: Voters were randomly selected.
10% condition: 630 and 1010 are both less than 10% of all voters.
Independent samples condition: Since the samples were random, the groups are independent.
Success/Failure condition: $n\hat{p}$ (before) = (630)(0.54) = 340, $n\hat{q}$ (before) = (630)(0.46) = 290, $n\hat{p}$ (after) = (1010)(0.51) = 515, and $n\hat{q}$ (after) = (1010)(0.49) = 505 are all greater than 10, so both samples are large enough.

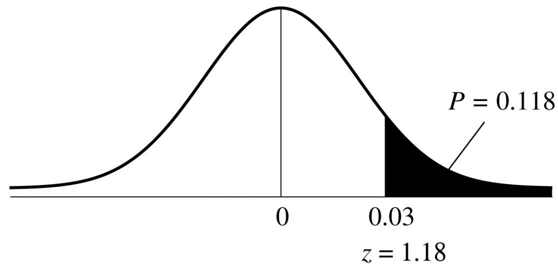
Since the conditions have been satisfied, we will model the sampling distribution of the difference in proportion with a Normal model with mean 0 and standard deviation estimated by:

$$SE_{\text{pooled}}\left(\hat{p}_B-\hat{p}_A\right)=\sqrt{\frac{\hat{p}_{\text{pooled}}\hat{q}_{\text{pooled}}}{n_B}+\frac{\hat{p}_{\text{pooled}}\hat{q}_{\text{pooled}}}{n_A}}=\sqrt{\frac{(0.5215)(0.4785)}{630}+\frac{(0.5215)(0.4785)}{1010}}\approx 0.02536.$$

The observed difference between the proportions is: 0.54 – 0.51 = 0.03.

Since the P-value = 0.118 is fairly high, we fail to reject the null hypothesis. There is little evidence of a decrease in the proportion of voters in support of the candidate after the news of his extramarital affair got out.

$$z = \frac{0.03 - 0}{0.02536}$$
$$z \approx 1.18$$



b) No evidence of a decrease in the proportion of voters in support of the candidate was found. If there is actually a decrease, and we failed to notice, that's a Type II error.

24. Shopping.

a) H_0 : The proportion of men who have purchased books online is the same as the proportion of women who have purchased books online. $(p_M = p_W \text{ or } p_M - p_W = 0)$

H_A : The proportion of men who have purchased books online is greater than the proportion of women who have purchased books online. $(p_M > p_W \text{ or } p_M - p_W > 0)$

Randomization condition: The men and women were chosen randomly.
10% condition: 222 and 208 are both less than 10% of all people.
Independent samples condition: The groups were chosen independently.
Success/Failure condition: $n\hat{p}$(men) = (222)(0.21) = 47, $n\hat{q}$(men) = (222)(0.79) = 175, $n\hat{p}$(women) = (208)(0.18) = 37, and $n\hat{q}$(women) = (208)(0.82) = 171 are all greater than 10, so both samples are large enough.

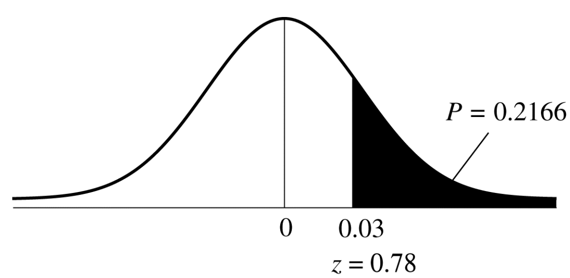
Since the conditions have been satisfied, we will model the sampling distribution of the difference in proportion with a Normal model with mean 0 and standard deviation estimated by:

$$SE_{\text{pooled}}(\hat{p}_M - \hat{p}_W) = \sqrt{\frac{\hat{p}_{\text{pooled}}\hat{q}_{\text{pooled}}}{n_M} + \frac{\hat{p}_{\text{pooled}}\hat{q}_{\text{pooled}}}{n_W}} = \sqrt{\frac{(0.1955)(8045)}{222} + \frac{(0.1955)(8045)}{208}} \approx 0.03827.$$

The observed difference between the proportions is:
0.21 – 0.18 = 0.03.

Since the P-value = 0.2166 is high, we fail to reject the null hypothesis. There is no evidence that the proportion of men who have purchased books online is greater than the proportion of women who have purchased books online.

$$z = \frac{0.03 - 0}{0.03827}$$
$$z \approx 0.78$$



b) If there is actually a difference between the proportions of men who have purchased books online and the proportion of women who have purchased books online, then we have committed a Type II error.

c) Most of the conditions were checked in part a. We only have one more to check:
Independent samples condition: There is no reason to believe that the samples of men and women influence each other in any way.

Since the conditions have been satisfied, we will find a two-proportion z-interval.

$$\left(\hat{p}_M - \hat{p}_W\right) \pm z^* \sqrt{\frac{\hat{p}_M\hat{q}_M}{n_M} + \frac{\hat{p}_W\hat{q}_W}{n_W}}$$

$$= \left(0.21 - 0.18\right) \pm 1.960\sqrt{\frac{(0.21)(0.79)}{222} + \frac{(0.18)(0.82)}{208}} = \left(-0.04, 0.11\right)$$

d) We are 95% confident that the proportion of men who buy books online could be 4 percentage points lower than and up to 11 percent higher than the proportion of women who buy books online.

25. Twins.

a) H_0: The proportion of multiple births is the same for white women and black women.
$\left(p_W = p_B \text{ or } p_W - p_B = 0\right)$

H_A: The proportion of multiple births is different for white women and black women.
$\left(p_W \neq p_B \text{ or } p_W - p_B \neq 0\right)$

Randomization condition: Assume that these women are representative of all women.
10% condition: 3132 and 606 are both less than 10% of all people.
Independent samples condition: The groups are independent.
Success/Failure condition: $n\hat{p}$(white) = 94, $n\hat{q}$(white) = 3038, $n\hat{p}$(black) = 20, and $n\hat{q}$(black) = 586 are all greater than 10, so both samples are large enough.

Since the conditions have been satisfied, we will model the sampling distribution of the difference in proportion with a Normal model with mean 0 and standard deviation estimated by:

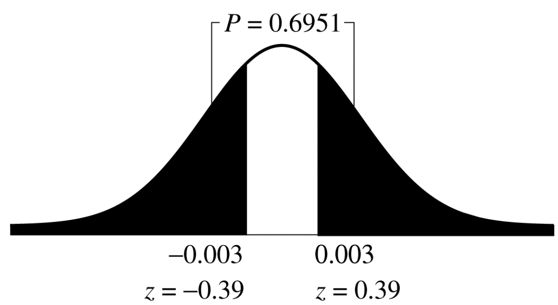
$$SE_{\text{pooled}}\left(\hat{p}_W - \hat{p}_B\right) = \sqrt{\frac{\hat{p}_{\text{pooled}}\hat{q}_{\text{pooled}}}{n_W} + \frac{\hat{p}_{\text{pooled}}\hat{q}_{\text{pooled}}}{n_B}} = \sqrt{\frac{\left(\frac{114}{3738}\right)\left(\frac{3624}{3738}\right)}{3132} + \frac{\left(\frac{114}{3738}\right)\left(\frac{3624}{3738}\right)}{606}} \approx 0.007631.$$

The observed difference between the proportions is:
0.030 – 0.033 = –0.003.

Since the P-value = 0.6951 is high, we fail to reject the null hypothesis. There is no evidence of a difference between the proportions of multiple births for white women and black women.

$$z = \frac{0.003 - 0}{0.07631}$$

$$z \approx -0.39$$



b) If there is actually a difference between the proportions of multiple births for white women and black women, then we have committed a Type II error.

26. Mammograms.

a) H_0: The proportion of deaths from breast cancer is the same for women who never had a mammogram as for women who had mammograms. $(p_N = p_M \text{ or } p_N - p_M = 0)$

H_A: The proportion of deaths from breast cancer is greater for women who never had a mammogram than for women who had mammograms. $(p_N > p_M \text{ or } p_N - p_M > 0)$

Randomization condition: Assume that the women are representative of all women.
10% condition: 21,195 and 21,088 are both less than 10% of all women.
Independent samples condition: The groups were chosen independently.
Success/Failure condition: $n\hat{p}$(never) = 66, $n\hat{q}$(never) = 21,129, $n\hat{p}$(mammogram) = 63, and $n\hat{q}$(mammogram) = 21,025 are all greater than 10, so both samples are large enough.

Since the conditions have been satisfied, we will model the sampling distribution of the difference in proportion with a Normal model with mean 0 and standard deviation estimated by:

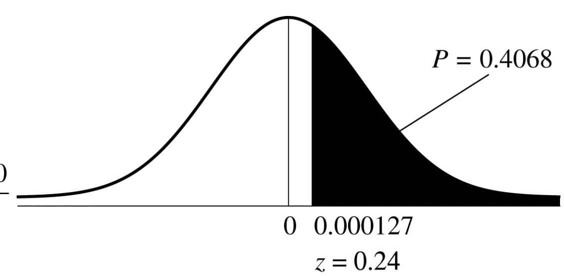
$$SE_{\text{pooled}}(\hat{p}_N - \hat{p}_M) = \sqrt{\frac{\hat{p}_{\text{pooled}}\hat{q}_{\text{pooled}}}{n_N} + \frac{\hat{p}_{\text{pooled}}\hat{q}_{\text{pooled}}}{n_M}} = \sqrt{\frac{\left(\frac{129}{42,283}\right)\left(\frac{42,154}{42,283}\right)}{21,195} + \frac{\left(\frac{129}{42,283}\right)\left(\frac{42,154}{42,283}\right)}{21,088}} \approx 0.000536.$$

The observed difference between the proportions is:
0.003114 – 0.002987 = 0.000127.

Since the P-value = 0.4068 is high, we fail to reject the null hypothesis. There is no evidence that the proportion of breast cancer deaths for women who have never had a mammogram is greater than the proportion of deaths from breast cancer for women who underwent screening by mammogram.

$$z = \frac{0.000127 - 0}{0.000536}$$

$$z \approx 0.24$$



b) If the proportion of deaths from breast cancer for women who have not had mammograms is actually greater than the proportion of deaths from breast cancer for women who have had mammograms, we have committed a Type II error.

27. Pain.

a) **Randomization condition:** The patients were randomly selected AND randomly assigned to treatment groups. If that's not random enough for you, I don't know what is!
10% condition: 112 and 108 are both less than 10% of all people with joint pain.
Success/Failure condition: $n\hat{p}$(A) = 84, $n\hat{q}$(A) = 28, $n\hat{p}$(B) = 66, and $n\hat{q}$(B) = 42 are all greater than 10, so both samples are large enough.

Since the conditions are met, we can use a one-proportion z-interval to estimate the percentage of patients who may get relief from medication A.

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}\hat{q}}{n}} = \left(\frac{84}{112}\right) \pm 1.960 \sqrt{\frac{\left(\frac{84}{112}\right)\left(\frac{28}{112}\right)}{112}} = (67.0\%, 83.0\%)$$

We are 95% confident that between 67.0% and 83.0% of patients with joint pain will find medication A to be effective.

b) Since the conditions were met in part a, we can use a one-proportion z-interval to estimate the percentage of patients who may get relief from medication B.

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}\hat{q}}{n}} = \left(\frac{66}{108}\right) \pm 1.960 \sqrt{\frac{\left(\frac{66}{108}\right)\left(\frac{42}{108}\right)}{108}} = (51.9\%, 70.3\%)$$

We are 95% confident that between 51.9% and 70.3% of patients with joint pain will find medication B to be effective.

c) The 95% confidence intervals overlap, which might lead one to believe that there is no evidence of a difference in the proportions of people who find each medication effective. However, if one was lead to believe that, one should proceed to part…

d) Most of the conditions were checked in part a. We only have one more to check:
Independent samples condition: The groups were assigned randomly, so there is no reason to believe there is a relationship between them.

Since the conditions have been satisfied, we will find a two-proportion z-interval.

$$(\hat{p}_A - \hat{p}_B) \pm z^* \sqrt{\frac{\hat{p}_A\hat{q}_A}{n_A} + \frac{\hat{p}_B\hat{q}_B}{n_B}}$$

$$= \left(\tfrac{84}{112} - \tfrac{66}{108}\right) \pm 1.960 \sqrt{\frac{\left(\frac{84}{112}\right)\left(\frac{28}{112}\right)}{112} + \frac{\left(\frac{66}{108}\right)\left(\frac{42}{108}\right)}{112}} = (0.017, 0.261)$$

We are 95% confident that the proportion of patients with joint pain who will find medication A effective is between 1.70% and 26.1% higher than the proportion of patients who will find medication B effective.

e) The interval does not contain zero. There is evidence that medication A is more effective than medication B.

f) The two-proportion method is the proper method. By attempting to use two, separate, confidence intervals, you are adding standard deviations when looking for a difference in proportions. We know from our previous studies that *variances* add when finding the standard deviation of a difference. The two-proportion method does this.

28. Gender gap.

a) **Randomization condition:** The poll was probably random, although not specifically stated.
10% condition: 473 and 522 are both less than 10% of all voters.
Success/Failure condition: $n\hat{p}$(men) = 246, $n\hat{q}$(men) = 227, $n\hat{p}$(women) = 235, and $n\hat{q}$(women) = 287 are all greater than 10, so both samples are large enough.

Since the conditions are met, we can use a one-proportion z-interval to estimate the percentage of men who may vote for the candidate.

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}\hat{q}}{n}} = (0.52) \pm 1.960\sqrt{\frac{(0.52)(0.48)}{473}} = (47.5\%, 56.5\%)$$

We are 95% confident that between 47.5% and 56.5% of men may vote for the candidate.

b) Since the conditions were met in part a, we can use a one-proportion *z*-interval to estimate the percentage of women who may vote for the candidate.

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}\hat{q}}{n}} = (0.45) \pm 1.960\sqrt{\frac{(0.45)(0.55)}{522}} = (40.7\%, 49.3\%)$$

We are 95% confident that between 40.7% and 49.3% of women may vote for the candidate.

c) The 95% confidence intervals overlap, which might make you think that there is no evidence of a difference in the proportions of men and women who may vote for the candidate. However, if you think that, don't delay! Move on to part…

d) Most of the conditions were checked in part a. We only have one more to check:
Independent samples condition: There is no reason to believe that the samples of men and women influence each other in any way.

Since the conditions have been satisfied, we will find a two-proportion *z*-interval.

$$\left(\hat{p}_M - \hat{p}_W\right) \pm z^* \sqrt{\frac{\hat{p}_M\hat{q}_M}{n_M} + \frac{\hat{p}_W\hat{q}_W}{n_W}}$$
$$= (0.52 - 0.45) \pm 1.960\sqrt{\frac{(0.52)(0.48)}{473} + \frac{(0.45)(0.55)}{522}} = (0.008, 0.132)$$

We are 95% confident that the proportion of men who may vote for the candidate is between 0.8% and 13.2% higher than the proportion of women who may vote for the candidate.

e) The interval does not contain zero. There is evidence that the proportion of men may vote for the candidate is greater than the proportion of women who may vote for the candidate.

f) The two-proportion method is the proper method. By attempting to use two, separate, confidence intervals, you are adding standard deviations when looking for a difference in proportions. We know from our previous studies that *variances* add when finding the standard deviation of a difference. The two-proportion method does this.

29. Sensitive men.

H_0: The proportion of 18-24-year-old men who are comfortable talking about their problems is the same as the proportion of 25-34-year old men.
$\left(p_{Young} = p_{Old} \text{ or } p_{Young} - p_{Old} = 0\right)$

H_A: The proportion of 18-24-year-old men who are comfortable talking about their problems is higher than the proportion of 25-34-year old men.
$\left(p_{Young} > p_{Old} \text{ or } p_{Young} - p_{Old} > 0\right)$

Randomization condition: We must assume that the respondents were chosen randomly.
10% condition: 129 and 184 are both less than 10% of all people.
Independent samples condition: The groups were chosen independently.
Success/Failure condition: $n\hat{p}$(young) = 80, $n\hat{q}$(young) = 49, $n\hat{p}$(old) = 98, and $n\hat{q}$(old) = 86 are all greater than 10, so both samples are large enough.

Since the conditions have been satisfied, we will model the sampling distribution of the difference in proportion with a Normal model with mean 0 and standard deviation estimated

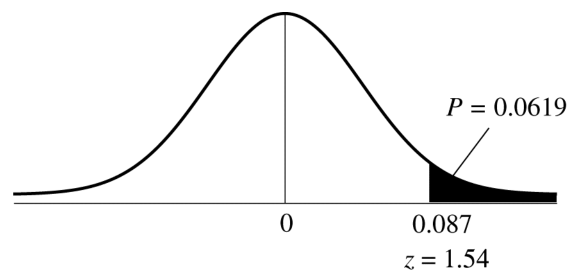
by:. $SE_{\text{pooled}}\left(\hat{p}_{Young} - \hat{p}_{Old}\right) = \sqrt{\dfrac{\hat{p}_{\text{pooled}}\hat{q}_{\text{pooled}}}{n_Y} + \dfrac{\hat{p}_{\text{pooled}}\hat{q}_{\text{pooled}}}{n_O}} = \sqrt{\dfrac{\left(\frac{178}{313}\right)\left(\frac{135}{313}\right)}{129} + \dfrac{\left(\frac{178}{313}\right)\left(\frac{135}{313}\right)}{184}} \approx 0.05687$

The observed difference between the proportions is:
0.620 – 0.533 = 0.087.

Since the *P*-value = 0.0619 is high, we fail to reject the null hypothesis. There is little evidence that the proportion of 18-24-year-old men who are comfortable talking about their problems is higher than the proportion of 25-34-year-old men who are comfortable. *Time* magazine's interpretation is questionable.

$z = \dfrac{0.087 - 0}{0.05687}$

$z \approx 1.54$



30. Retention rates.

H_0: The retention rates for private colleges is the same as the retention rate for public colleges. $\left(p_{Private} = p_{Public} \text{ or } p_{Private} - p_{Public} = 0\right)$

H_A: The retention rates for private colleges is the different from the retention rate for public colleges. $\left(p_{Private} \neq p_{Public} \text{ or } p_{Private} - p_{Public} \neq 0\right)$

Randomization condition: Assume that the samples were random.
10% condition: 1139 and 505 are both less than 10% of all colleges.
Independent samples condition: The samples were taken independently.
Success/Failure condition: $n\hat{p}$(private) = 853, $n\hat{q}$(private) = 286, $n\hat{p}$(public) = 363, and $n\hat{q}$(public) = 142 are all greater than 10, so both samples are large enough.

Since the conditions have been satisfied, we will model the sampling distribution of the difference in proportion with a Normal model with mean 0 and standard deviation estimated by:

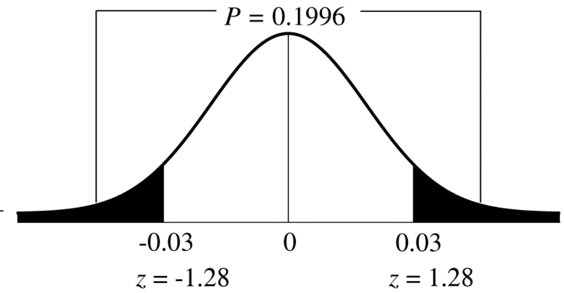
$$SE_{\text{pooled}}\left(\hat{p}_{Pri} - \hat{p}_{Pub}\right) = \sqrt{\frac{\hat{p}_{\text{pool}}\hat{q}_{\text{pool}}}{n_{Private}} + \frac{\hat{p}_{\text{pool}}\hat{q}_{\text{pool}}}{n_{Public}}} = \sqrt{\frac{(0.7397)(0.2603)}{1139} + \frac{(0.7397)(0.2603)}{505}} \approx 0.02346.$$

The observed difference between the proportions is:
0.749 – 0.719 = 0.03.

Since the P-value = 0.1996 is high, we fail to reject the null hypothesis. There is no evidence that the retention rates are different at private and public colleges.

$$z = \frac{0.03 - 0}{0.02346}$$

$$z \approx 1.28$$



31. Online activity checks.

H_0: The proportion of teens who say their parents check to see what websites they visited is the same in 2006 as it was in 2004. $\left(p_{2006} = p_{2004} \text{ or } p_{2006} - p_{2004} = 0\right)$

H_A: The proportion of teens who say their parents check to see what websites they visited is higher in 2006 than it was in 2004. $\left(p_{2006} > p_{2004} \text{ or } p_{2006} - p_{2004} > 0\right)$

Randomization condition: The samples were random.
10% condition: 811 and 868 are both less than 10% of all teens.
Independent samples condition: The samples were taken independently.
Success/Failure condition: $n\hat{p}$(2006) = 333, $n\hat{q}$(2006) = 478, $n\hat{p}$(2004) = 286, and $n\hat{q}$(2004) = 582 are all greater than 10, so both samples are large enough.

Since the conditions have been satisfied, we will model the sampling distribution of the difference in proportion with a Normal model with mean 0 and standard deviation estimated by:

$$SE_{\text{pooled}}\left(\hat{p}_{2006} - \hat{p}_{2004}\right) = \sqrt{\frac{\hat{p}_{\text{pool}}\hat{q}_{\text{pool}}}{n_{2006}} + \frac{\hat{p}_{\text{pool}}\hat{q}_{\text{pool}}}{n_{2004}}} = \sqrt{\frac{(0.369)(0.631)}{811} + \frac{(0.369)(0.631)}{868}} \approx 0.02355.$$

The observed difference between the proportions is: 0.41 – 0.33 = 0.08. We will perform a 2-proportion z-test.

The value of z = 3.44 and the P-value = 0.0003. Since the P-value is low, we reject the null hypothesis. There is strong evidence that a greater proportion of teens in 2006 say their parents checked in to see what web sites they visited than said this in 2004.

32. Computer gaming.

H_0: The proportion of gamers the same for boys aged 12-14 as it is for boys aged 15-17.
$\left(p_{12-14} = p_{15-17} \text{ or } p_{12-14} - p_{15-17} = 0\right)$

H_A: The proportion of gamers different for boys aged 12-14 than it is for boys aged 15-17.
$\left(p_{12-14} \neq p_{15-17} \text{ or } p_{12-14} - p_{15-17} \neq 0\right)$

Randomization condition: Assume that the samples were random.
10% condition: 223 and 248 are both less than 10% of all boys.
Independent samples condition: The samples were taken independently.
Success/Failure condition: $n\hat{p}$(12-14) = 154, $n\hat{q}$(12-14) = 69, $n\hat{p}$(15-17) = 154, and $n\hat{q}$(15-17) = 94 are all greater than 10, so both samples are large enough.

Since the conditions have been satisfied, we will model the sampling distribution of the difference in proportion with a Normal model with mean 0 and standard deviation estimated by:

$$SE_{\text{pooled}}\left(\hat{p}_{12-14} - \hat{p}_{15-17}\right) = \sqrt{\frac{\hat{p}_{\text{pool}}\hat{q}_{\text{pool}}}{n_{12-14}} + \frac{\hat{p}_{\text{pool}}\hat{q}_{\text{pool}}}{n_{15-17}}} = \sqrt{\frac{(0.654)(0.346)}{223} + \frac{(0.654)(0.346)}{248}} \approx 0.0439 .$$

The observed difference between the proportions is: 0.69 – 0.62 = 0.07. We will perform a 2 proportion z-test.

Since the P-value = 0.11 is high, we fail to reject the null hypothesis. There is no evidence that the proportion of boys aged 12-14 who play computer games is any different than the proportion of boys aged 15-17 who play video games.

$$z = \frac{0.07 - 0}{0.0439}$$

$z \approx 1.59$